Radiative decay $B \to l\nu\gamma$ in the light cone QCD approach

G. Eilam , I. Halperin , R.R. Mendel[*]

Technion - Israel Institute of Technology
Department of Physics
Haifa, 32000, Israel
e-mail addresses: phr82ge@technion.technion.ac.il
higor@techunix.technion.ac.il
mendel@uwo.ca

Abstract:

We calculate the rate for the decay $B_u \to l\nu\gamma$ using the light cone QCD sume rules. We find $Br(B_u \to l\nu\gamma) \simeq 2 \cdot 10^{-6}$. The results are used to test the applicability of the constituent quark model approximation to the same process. The latter estimate is proportional to $1/m_u^2$, where $m_u \simeq \bar{\Lambda}_u$ is the "constituent quark mass", indicating that the process is of long distance type. We find that the two approaches yield similar results for the total rate with the choice $m_u \simeq 480~MeV$. This indicates that the constituent quark model may be used for estimates of the radiative "annihilation" contribution to this and other radiative decays. We point out that this decay may be useful for the measurement of $|V_{ub}|$.

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[*] On sabbatical leave from Dept. of Applied Math., Univ. of Western Ontario, London, Ontario, Canada

1 Introduction

Recently there has been increasing interest in long distance contributions to inclusive [1, 2, 3] and exclusive [4, 5, 6, 7, 8] radiative D- and B-decays. One way of estimating the long distance part of annihilation-type contributions is to use a simple constituent quark model in which the photon (or gluon) is emitted from the light quark in the initial state [1, 2, 4, 6, 7, 8]. Thus for radiative decays such as $B \to l\nu\gamma$ or $B \to \rho\gamma$ one finds that the annihilation amplitude is proportional to $1/m_q$, where q is the light quark in the decaying meson. This indicates that it is indeed of the long distance type in an average sense, i.e. it is equivalent to a sum of all possible hadronic intermediate states [5]. One expects that more detailed observables, such as the photon spectrum in $B \to l\nu\gamma$, will not be reproduced in full detail by the quark model.

In a recent paper [7] the reaction $B \to l\nu\gamma$ was discussed using the annihilation static quark diagram with the photon emitted from the u-quark giving the largest contribution. The result of this calculation exhibits the particular $1/m_u^2$ dependence (see Eq. (25) below). One may expect that in the case at hand this constituent quark mass m_u must be close to the "inertia parameter" $\bar{\Lambda}_u$ of the Heavy Quark Effective Theory, which is defined as $\bar{\Lambda} = \lim_{m_Q \to \infty}(M_{(\bar{Q}q)} - m_Q)$. Estimates of $\bar{\Lambda}_u$ typically yield a somewhat larger value for this quantity than the number $m_u \simeq 350~MeV$, which holds for light hadrons (see e.g. [9]).

In this letter we suggest a different and largely model-independent method to calculate the differential and total width of the decay $B \to l\nu\gamma$. Our approach is based on the use of the concept of duality in a form suggested by the QCD sum rules method [10]. In this approach an amplitude of interest is extracted from the imaginary part of a suitably defined Euclidean correlation function. More specifically, we use here a version of the QCD sum rule technique known as the light cone QCD sum rules. The method has been developed for light quark systems in Ref. [11],[12]. It has proved to be a convenient tool for a study of exclusive processes with the emission of a light particle. An Euclidean correlation function corresponding to such a process can be calculated using the light final particle as a source of a properly defined variable external field in which the correlation function develops. It turns out that in this case different operators contribute according to their twist rather than their dimension. Matrix elements of non-local operators in the variable external field are identified with the set of wave functions of increasing twist, and replace vacuum expectation values of local operators which appear in the traditional sum rules method. The behavior of wave functions is severely restricted by the (approximate) conformal invariance of QCD. More details on the method and a list of references can be found in [13], [14], see also Sect. 2 and 3 below.

In our problem we deal with a set of the photon wave functions of increasing twist. Photon matrix elements are defined via a sum of matrix elements containing vector

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meson states (see sect.3 for more details). Unlike perhaps more complicated hadron (such as π- or ρ-) wave functions, which are still debatable in the literature, the photon wave functions turn out to be rather simple and are given by their asymptotic expressions. Thus theoretical uncertainties are minimal in this case. We note in passing that our approach in not equivalent to the direct use of the vector meson dominance applied in [6] to compare with the quark model annihilation contribution in the decay $B \to \rho\gamma$. The paper is organized as follows. In sect.2 we obtain the light cone sum rule for the process of interest to the twist 2 accuracy (with partial account for twist 4 effects). In sect.3 we collect the necessary information on the photon wave functions of twist 2. The final sum rule and a numerical analysis are presented in sect.4. We find transition form factors and calculate the differential and total decay width. The results are then confronted with those of Ref. [7]. We find that the rates obtained by these two methods are the same for the values of the constituent quark mass $m_u \simeq 480\ MeV$. This range is in reasonable agreement with independent estimates for $\bar{\Lambda}$. These results indicate that the quark model approximation may be used for estimates of the radiative "annihilation" contribution to this and other heavy meson decays, in which the photon is dominantly emitted from the light quark, such as $\bar{B}^0 \to D^{0\star}\gamma$, $B^\pm \to \rho^\pm\gamma$, $B_s \to \gamma\gamma$. Sect. 5 contains a summary and several concluding remarks. We finally comment on the possible usefulness of this decay for the measurement of $|V_{ub}|$ and discuss some general problems related to the annihilation mechanism in exclusive amplitudes.

2 Light cone sum rule

The effective Hamiltonian for the decay of interest is

$$H_{eff} = \frac{G_F}{\sqrt{2}}V_{ub}\ \bar{\nu}\gamma_\mu(1 - \gamma_5)l\ \langle\gamma|\bar{u}\gamma_\mu(1 - \gamma_5)b|B\rangle \tag{1}$$

We have neglected here the helicity-suppressed photon emission from the final lepton. We use the following parametrization for the hadron transition form factors

$$
\begin{aligned}
\frac{1}{\sqrt{4\pi\alpha}}\langle\gamma(q)|\bar{u}\Gamma_\mu b|B(p)\rangle &= \varepsilon_{\mu\nu\rho\sigma}e^*_\nu p_\rho q_\sigma\frac{g(Q^2)}{m_B^2} \\
&+ i(e^*_\mu(pq) - (e^*p)q_\mu)\frac{f(Q^2)}{m_B^2}
\end{aligned} \tag{2}
$$

(here $\Gamma_\mu = \gamma_\mu(1 - \gamma_5)$, $Q = p - q$ and e^*_μ stands for the polarization vector of the photon.) Our aim is to calculate the transition form factors g , f including their momentum dependence. To this end we use the light cone QCD sum rules method. Technically, our method is very close to the calculation of the semileptonic $B \to \pi e\nu$ [15] and radiative $B \to \rho + \gamma$, $B \to K^\star + \gamma$ [16] form factors. These calculations are based on using the light cone wave functions of a light pseudoscalar (π) or vector

($\rho, K^\star$) meson, respectively. In our problem, the relevant objects are the light cone photon wave functions, which will be discussed below.

We start with the correlation function

$$
\begin{aligned}
T_\mu(p, q) &= i \int d^4 x \, e^{ipx} \langle \gamma(q)|T\{\bar{u}(x)\gamma_\mu(1-\gamma_5)b(x)\bar{b}(0)i\gamma_5 u(0)\}|0\rangle \\
&= \frac{m_B^2 f_B}{m_b} \frac{1}{m_B^2 - (p+q)^2} \langle \gamma(q)|\bar{u}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle + \ldots \quad (3) \\
&= \sqrt{4\pi\alpha}\left\{\varepsilon_{\mu\nu\rho\sigma}e_\nu^* p_\rho q_\sigma T_1 + i(e_\mu^*(pq) - (e^*p)q_\mu)T_2\right\}
\end{aligned}
$$

where $\langle B|\bar{b}i\gamma_5 u|0\rangle = m_B^2 f_B/m_b$. The second line in the above formula selects the pole contribution due to the B-meson pole in the corresponding dispersion integrals for the scalar form factors g and f. On the other hand, the correlation function (3) can be calculated in QCD at large Euclidean momenta $(p+q)^2$.

In this case the virtuality of the heavy quark, which is of order $m_b^2 - (p+q)^2$, is the large quantity. Thus, we can expand the heavy quark propagator in powers of slowly varying fields residing in the photon, which act as external fields on the propagating heavy quark. The expansion in powers of an external field is also the expansion of the propagator in powers of a deviation from the light cone $x^2 \simeq 0$. The leading contribution is obtained by using the free heavy quark propagator in the correlation function (3). We obtain

$$
T_\mu(p, q) = \int \frac{d^4x d^4k}{(2\pi)^4(m_b^2 - k^2)} e^{i(p-k)x}\langle \gamma(q)|\bar{u}(x)\gamma_\mu(1-\gamma_5)(k_\mu\gamma_\mu + m_b)i\gamma_5 u(0)|0\rangle \quad (4)
$$

In this formula a path-ordered gauge factor between the quark fields is implied, as required by gauge invariance. In the particular case of the Fock-Schwinger gauge $x_\mu A_\mu(x) = 0$ this factor is equal to unity.

We see that the answer is expressed via the one-photon matrix element of the gauge invariant non-local operator with a light-like separation $x^2 \simeq 0$. This matrix element defines a light cone photon wave function (WF) in a way analogous to the definition of the light cone meson WF's [17]. We define two-particle photon WF's as follows :

$$
\begin{aligned}
\langle \gamma(q)|\bar{u}(x)\sigma_{\mu\nu}u(0)|0\rangle &= i\sqrt{4\pi\alpha}e_u\langle\bar{u}u\rangle\int_0^1 du \, e^{iuqx}[(e_\mu q_\nu - e_\nu q_\mu)(\chi\phi(u) \\
&+ x^2 g^{(1)}(u)) + \{(qx)(e_\mu x_\nu - e_\nu x_\mu) + (ex)(x_\mu q_\nu - x_\nu q_\mu) \\
&- x^2(e_\mu q_\nu - e_\nu q_\mu)\}g^{(2)}(u)] \quad (5) \\
\langle \gamma(q)|\bar{u}(x)\gamma_\mu\gamma_5 u(0)|0\rangle &= \frac{1}{4}\sqrt{4\pi\alpha}\varepsilon_{\mu\nu\rho\sigma}e_\nu q_\rho x_\sigma f \int_0^1 du \, e^{iuqx}g_\perp(u) \quad (6)
\end{aligned}
$$

Here $\phi(u)$, $g_\perp(u)$ stand for the leading twist 2 photon WF, while $g^{(1)}$ and $g^{(2)}$ are the two-particle WF's of twist 4 (see below for more detail). The above WF's describe

a distribution in the fraction of the total momentum carried by the quark (up_z) and the anti-quark $((1-u)p_z)$ in the infinite-momentum frame $p_z \to \infty$ (or, equivalently, a distribution in the light cone momentum P_+). The dimensional constants χ and f are chosen in such a way that a function $f(u) = (\phi(u), g_\perp(u))$ is normalized to unity :

$$\int_0^1 du\, f(u) = 1 \tag{7}$$

It is assumed that perturbative logs of x^2 are resummed by renormalization group methods and result in a dependence of the WF's on the scale factor $\mu^2 \sim x^{-2} \sim m_b^2 - (p+q)^2$, which in other words fixes the normalization point of local operators arising in an expansion of the non-local matrix elements (5-7). The corresponding anomalous dimensions can be found in the literature.

The WF's $g^{(1)}$, $g^{(2)}$ represent twist 4 contributions to the two-particle photon WF (5). Using equations of motion one can relate them with three paricle WF's of twist 4 which include an additional gluon [11, 12]. These three-particle WF's would appear in the correlation function (3) if a soft gluon emission from the heavy quark is taken into account. Such a contribution is typically rather small and will be omitted in what follows. On the other hand, we retain the two-particle twist 4 contribution due to $g^{(1)}$, $g^{(2)}$. That is, the most important twist 4 effects are included in our calculation. Finally, adding a perturbative photon emission from the u- and b-quarks we arrive at the following expressions for the invariant functions T_1, T_2 :

$$
\begin{aligned}
T_1 &= \int_0^1 \frac{1}{m_b^2 - (p+uq)^2} \left[e_u \langle \bar u u \rangle \left(\chi \phi(u) - 4 \frac{g^{(1)}(u) - g^{(2)}(u)}{m_b^2 - (p+uq)^2} (1 + \frac{2m_b^2}{m_b^2 - (p+uq)^2}) \right) \right. \\
&\quad + \left. \frac{m_b}{2} f \frac{g_\perp(u)}{m_b^2 - (p+uq)^2} + \frac{3m_b}{4\pi^2} \left((e_u - e_b) \bar u \frac{m_b^2 - p^2}{m_b^2 - \bar u p^2} + e_b \ln \frac{m_b^2 - \bar u p^2}{u m_b^2} \right) \right]
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
T_2 &= \int_0^1 \frac{1}{m_b^2 - (p+uq)^2} \left[e_u \langle \bar u u \rangle \left(\chi \phi(u) - 4 \frac{g^{(1)}(u)}{m_b^2 - (p+uq)^2} (1 + \frac{2m_b^2}{m_b^2 - (p+uq)^2}) \right) \right. \\
&\quad + \frac{3m_b^3}{4\pi^2(m_b^2 - p^2)} \left\{ \left((e_u - e_b)(u - \bar u + \frac{p^2}{m_b^2} - \frac{p^2 u^2}{m_b^2 - \bar u p^2}) - (e_u + e_b) u \frac{p^2}{m_b^2} \right) \frac{\bar u(m_b^2 - p^2)}{m_b^2 - \bar u p^2} \right. \\
&\quad + \left. \left. e_b (u - \bar u + \frac{p^2}{m_b^2}) \ln \frac{m_b^2 - \bar u p^2}{u m_b^2} \right\} \right]
\end{aligned} \tag{9}
$$

We now proceed to a description of the photon WF's (5-6).

3   Light cone photon wave functions

The leading twist photon WF $\phi(u, \mu^2)$ has been introduced and investigated in detail in Ref.[11] in connection with a study of the radiative decay $\Sigma \to p + \gamma$. Formally, the photon matrix element of an arbitrary (non-local) operator is defined as a weighted

sum of matrix elements of the same operator between the vacuum and (transversely polarized) vector meson states :

$$
\begin{aligned}
\langle \gamma(q) | \ldots | 0 \rangle &= i\sqrt{4\pi\alpha}\, e_\mu^{(\lambda)} \int dz \; e^{iqz} \langle 0 | T j_\mu^{em}(z) \ldots | 0 \rangle \\
&= \sqrt{4\pi\alpha} \sum_n \frac{1}{g_n} \langle V_n(q) | \ldots | 0 \rangle \; ,
\end{aligned}
\tag{10}
$$

where $j_\mu^{em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$ and the couplings g_n are defined via

$$
\langle 0 | j_\mu^{em} | V_n \rangle = e_\mu^{(\lambda)} \frac{m_n^2}{g_n} \quad , \quad g_\rho \simeq 5.5
\tag{11}
$$

The particular value for g_ρ quoted above has been found long ago by the QCD sum rules method [10].

Our definition (10) is equivalent to the one given in [11], [12] for the case of the leading twist photon WF $\phi_\perp(u)$. There it has been defined as the vacuum expectation value of the non-local operator $\bar{q}\sigma_{\mu\nu}q$ (where $q = u, d, s$) in an external electromagnetic field $F_{\alpha\beta}(x)$. The QCD action is modified in this case to $\int dz\, L_{QCD}(z) + \sqrt{4\pi\alpha} \int dz\, A_\mu(z) j_\mu^{em}(z)$ and, expanding to the first order in the plane wave field strength $F_{\alpha\beta}(x) = i(e_\beta^{(\lambda)} q_\alpha - e_\alpha^{(\lambda)} q_\beta) e^{iqx}$, we obtain

$$
\int dz \; e^{iqx} \langle 0 | T\{ j_\mu^{em}(z) \bar{q}(x) \sigma_{\alpha\beta} q(0) \} | 0 \rangle = e_q \langle \bar{q}q \rangle \chi (q_\beta \delta_{\alpha\mu} - q_\alpha \delta_{\beta\mu}) \int du \; e^{iuqx} \phi(u) \tag{12}
$$

Thus, according to Eq.(10), one gets

$$
\langle \gamma(q) | \bar{u}(x) \sigma_{\alpha\beta} u(0) | 0 \rangle = i\sqrt{4\pi\alpha} e_u \langle \bar{u}u \rangle \chi (e_\alpha^{(\lambda)} q_\beta - e_\beta^{(\lambda)} q_\alpha) \int du \; e^{iuqx} \phi(u) \; ,
\tag{13}
$$

i.e. Eq.(5). In the constant external field limit $q \to 0$, Eq.(12) goes back to the definition of a quantity known as the magnetic susceptibility of the quark condensate χ[19]:

$$
\langle \bar{q}\sigma_{\alpha\beta}q \rangle_F = e_q \langle \bar{q}q \rangle \chi F_{\alpha\beta}
\tag{14}
$$

The magnetic susceptibility χ has been found [20], [21] by the QCD sum rules method. At an accuracy of the order of 30 % it is dominated by the ρ-meson contribution : at $\mu^2 \simeq 1\ GeV^2$

$$
\chi = \begin{cases} -4.4\ Gev^{-2} & \rho, \rho', \rho'' \text{ are included} \\ -3.3\ Gev^{-2} & \rho \text{ is included} \end{cases}
$$

It has been suggested on these grounds that the vector dominance works sufficiently well for electromagnetic properties of hadrons in the long wave limit $q \to 0$. Thus, to the quoted accuracy, by virtue of the local version of Eq.(10) we obtain the following value for the normalization constant f in Eq. (6) :

$$
f \simeq \frac{e_u}{g_\rho} f_\rho m_\rho \quad , \quad f_\rho \simeq 200\ MeV
\tag{15}
$$

The validity of Eq.(15) can be checked by the standard QCD sum rules technique.

Here some comments are in order. It is known that two-particle operators containing γ_μ and $\gamma_\mu\gamma_5$ instead of $\sigma_{\mu\nu}$ have non-zero projections on a vector meson state. The corresponding wave functions (call them $g_\perp^v$ and $g_\perp$, respectively) contain pieces of different twist. As has been shown in Ref. [16], a twist 2 contribution to a vector meson WF $g_\perp^v$ can be expressed via an integral of a longitudinally polarized vector meson WF of the leading twist 2. A similar relation holds for the derivative $dg_\perp(u)/du$ (see Eq.(39) of [16]). Thus we conclude that in the case of photon the WF $g_\perp^v$ vanishes to the leading twist accuracy, while the WF $g_\perp^v$ is a constant which is set to be unity with our normalization. At this point we differ from similar calculations in Refs. [23, 24] where a contribution due to $g_\perp$ was omitted. Numerically, the effect of the inclusion of the corresponding term in the sum rule (see Eq. (19) below) is about 25 % for the decay width.

Let us now discuss the much less trivial problem of finding the functional u-dependence of the WF's (5-6). As a non-local operator is equivalent to an infinite series of local operators , this task might appear formidable. Indeed, in order to restore the functional u-dependence, one has to know all moments $\int_0^1 du\ (2u-1)^n f(u)$, i.e. , according to (5-6), all matrix elements of the type $\langle\gamma|\bar{q}(\overleftrightarrow{\nabla}_\alpha\ x_\alpha)^n\sigma_{\mu\nu}q|0\rangle$. The way out lies in the use of an approximate conformal symmetry of QCD which holds at the one-loop level. The conformal invariance permits an expansion of a non-local operator of the type (5-6) in a series over multiplicatively renormalizable operators with ordered anomalous dimensions, which induces the expansion of the leading twist WF in the series of Gegenbauer polynomials [11],[12],[18]. The asymptotic WF's are defined as contributions of operators with the lowest conformal spin and unambiguously fixed by the group structure. Pre-asymptotic corrections correspond to the operators with the next-to-leading conformal spin, whose numerical values are usually calculated by the QCD sum rules method. It was found [11] that for the leading twist photon WF contributions of higher conformal spins are small. In terms of the dispersion relation this means that the corrections to the asymptotic WF's of the ρ,ρ',ρ'' states have opposite signs and nearly cancel in the sum. Thus, the simple asymptotic formula holds (in what follows, we use $\bar{u}\equiv 1-u$)

$$\phi_\perp(u) = 6u\bar{u} \tag{16}$$

To the leading twist accuracy we use for the WF (6) $g_\perp(u) = 1$. For the twist-4 WF's $g^{(1)}(u)$, $g^{(2)}(u)$ we use the following expressions (see [23]):

$$
\begin{aligned}
g^{(1)}(u) &= -\frac{1}{8}\bar{u}(3-u) \\
g^{(2)}(u) &= -\frac{1}{4}\bar{u}^2
\end{aligned}
\tag{17}
$$

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4 Form factors and the decay width

To match the answers (8,9) with the B-meson contribution to the correlation function (3), we note that (8,9) can be re-written as the dispersion integrals with the expression $(m_b^2 - \bar{u}p^2)/u$ being the mass of the intermediate state. The duality prescription requires that this invariant mass has to be restricted from above by the duality threshold $s_0 \simeq 35 \ GeV^2$ (this value is obtained from corresponding two-point sum rules). As it is easy to see, this transforms into an effective cut-off in the lower limit of the u-integral [15],[13]. Finally, we make the standard Borel transformation suppressing both higher states resonances and higher Fock states in the full photon wave function. Under the Borel transformation $-(p+q)^2 \to M^2$

$$\frac{1}{m_B^2 - (p+q)^2} \to \exp\left(-\frac{m_B^2}{M^2}\right)$$

$$\frac{1}{m_b^2 - (p+uq)^2} \to \frac{1}{u}\exp\left(-\frac{m_b^2 - \bar{u}p^2}{uM^2}\right) \qquad (18)$$

Our final sum rules take the form

$$
\begin{aligned}
g(p^2) &= \frac{m_b}{f_B}\int_0^1 \frac{du}{u}\ \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}p^2}{uM^2}\right)\Theta\left(u - \frac{m_b^2 - p^2}{s_0 - p^2}\right)\times \\
&\quad \left[e_u\langle\bar{u}u\rangle\left(\chi\phi(u) - 4(g^{(1)}(u) - g^{(2)}(u))\frac{m_b^2 + uM^2}{u^2 M^4}\right) + \frac{m_b f}{2uM^2}g_\perp(u)\right. \\
&\quad + \left. \frac{3m_b}{4\pi^2}\left\{(e_u - e_b)\bar{u}\frac{m_b^2 - p^2}{m_b^2 - \bar{u}p^2} + e_b \ln\frac{m_b^2 - \bar{u}p^2}{um_b^2}\right\}\right] \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
f(p^2) &= \frac{m_b}{f_B}\int_0^1 \frac{du}{u}\ \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}p^2}{uM^2}\right)\Theta\left(u - \frac{m_b^2 - p^2}{s_0 - p^2}\right)\times \\
&\quad \left[e_u\langle\bar{u}u\rangle\left(\chi\phi(u) - 4g^{(1)}(u)\frac{m_b^2 + uM^2}{u^2 M^4}\right)\right. \\
&\quad + \frac{3m_b^3}{4\pi^2(m_b^2 - p^2)}\left\{\left((e_u - e_b)(u - \bar{u} + \frac{p^2}{m_b^2} - \frac{p^2 u^2}{m_b^2 - \bar{u}p^2})\right.\right. \\
&\quad - \left.\left.\left.(e_u + e_b)u\frac{p^2}{m_b^2}\right)\frac{\bar{u}(m_b^2 - p^2)}{m_b^2 - \bar{u}p^2} + e_b(u - \bar{u} + \frac{p^2}{m_b^2})\ln\frac{m_b^2 - \bar{u}p^2}{um_b^2}\right\}\right] \qquad (20)
\end{aligned}
$$

We now turn to numerical estimates. In evaluating (19,20), we have used the following set of parameters : $m_b = 4.7 \ GeV$, $m_B = 5.28 \ GeV$, $s_0 \simeq 35 \ GeV^2$, $f_B \simeq 135 \ MeV$ [15],[14]. The particular value of f_B corresponds to the result of the sum rule calculation with $O(\alpha_s)$ corrections omitted, cf. [15], [14]. Within our accuracy we also neglect the logarithmic evolution of the WF's to a higher normalization point in the sum rules (19,20) which is of order of the characteristic virtuality of the heavy quark in the B-meson. The Borel mass M^2 has been varied in the interval from 8 to 20 GeV^2. We have found that within the variation of M^2 in this region, the result changes by less than 10 %. As the sum rules of the type (19,20) are

expected to work in the region $m_b^2 - p^2 \sim$ a few GeV^2, which is somewhat smaller than the maximal available $p^2 = m_b^2$, we have to use some extrapolation formulas to extent the results of the sum rules to the whole region of p^2. We have found that the best agreement is reached with the dipole formulas

$$g(p^2) \simeq \frac{h_1}{(1 - \frac{p^2}{m_1^2})^2} \quad , \quad f(p^2) \simeq \frac{h_2}{(1 - \frac{p^2}{m_2^2})^2} \tag{21}$$

The fact that the dipole approximation agrees better with results of the sum rules at not too large p^2 than the pole-type formulas has been also noted in [16] for analogous sum rules including vector mesons. We find

$$h_1 \simeq 1 \; GeV \quad , \quad m_1 \simeq 5.6 \; GeV$$
$$h_2 \simeq 0.8 \; GeV \quad , \quad m_2 \simeq 6.5 \; GeV \tag{22}$$

The calculation of the decay width yields

$$\Gamma = \frac{\alpha G_F^2 |V_{ub}|^2 m_B^5}{96\pi^2} I \tag{23}$$

where $(\bar{x} = 1 - x)$

$$m_B^2 I = \int_0^1 dx \, \bar{x}^3 x \{ g^2(x) + f^2(x) \} \tag{24}$$

Here $x = 1 - 2E/m_B$ and E stands for the photon energy. The corresponding differential width can be read off Eqs. (23,24). We expect the accuracy for I to be of the order of 20-30 %. The numerical answer for the quantity I is $I \simeq 0.021$ ($I \simeq 0.015$ if the contribution due to $g_\perp$ is omitted).

We would like now to compare our result with that obtained in Ref. [7] within the non-relativistic quark model. These authors have obtained the following answer for the decay width :

$$\Gamma = \frac{\alpha G_F^2 |V_{ub}|^2 m_B^5}{48\pi^2} \frac{Q_u^2}{6} \frac{f_B^2}{m_u^2} \; , \tag{25}$$

where now f_B stands for the physical value $f_B \simeq 175 \; MeV$ and a small contribution stemming from photon emission off the b-quark has been neglected. Note that in Eq. (23) the constant f_B appears implicitly in the denominator (see Eqs. (19,20)), while it is in the numerator of Eq.(25). This difference stems from the use of the dispersion approach. We find that Eq.(25) yields a result numerically close to (23) with the choice of the free parameter

$$m_u \simeq 480 \; MeV \; , \tag{26}$$

The photon spectra corresponding to the light cone QCD calculation and the quark model approach are compared in Fig.1 using the same normalization. One sees that while the quark model gives a fully symmetric spectrum, in our approach the spectrum is slightly asymmetric, as a result of a balance between a typical highly

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asymmetric resonance-type behaviour given by the non-perturbative contributions, and a perturbative photon emission. Both curves emphasize the dominant role of hard photons in the decay of interest and vanish at the end-points.

Finally, we present the estimated rate for the decay of interest

$$Br(B \to l\nu\gamma) \simeq 2 \cdot 10^{-6} \ , \tag{27}$$

which corresponds to the choice $m_u = 480 \ MeV$. We have used here the values $|V_{ub}|/|V_{cb}| = 0.08$, $|V_{cb}| = 0.04$ [22].

5   Summary

We have calculated within the light cone QCD sum rules technique the transition form factors and the differential and total width of the radiative decay $B \to l\nu\gamma$. In accordance with the expectations of Ref. [7], the photon emission overcomes the helicity suppression of the pure leptonic $B \to l\nu$ amplitude and yields an experimentally admissible decay width. Comparing our result with that suggested by the quark model, we have fixed the value of the effective constituent quark mass $m_u \simeq 480 \ MeV$, which is consistent with estimates for the "inertia parameter" $\bar{\Lambda}_u$ in the heavy quark effective theory. The photon spectra calculated within our approach and the quark model calculation have similar shapes. They both emphasize hard photon emission, vanish at the boundaries of the spectrum and do not exhibit the typical bremsstrahlung $1/E$ behavior. The completely symmetric form given by the quark model [7] is an artifact of the absence of hadronic poles. The light cone QCD approach gives a slightly asymmetric spectrum due to a balance between resonance-type non-perturbative contributions and perturbative photon emission.

In the perturbative QCD approach to exclusive processes [17] annihilation amplitudes are typically expressed via a convolution of wave functions of participating hadrons. These integrals turn out to be logarithmically divergent in the end-point region [17], which indicates the breakdown of perturbative factorization and suggests that the annihilation mechanism is of long distance type. This phenomenon is a manifestation of a long distance dominance, akin to an appearence of the $1/m_q$ factor in the quark model approach. In this sense the situation is similar to the problem of the Feynman-like end-point contribution to the pion form factor [13]. We believe that the dispersion light cone QCD approach, suggested in this context in Ref. [13] and used in the present letter for the particular annihilation-like process, may be of use also for reactions involving gluon annihilation.

We conclude by remarking that this process may be useful to extact the KM matrix element $|V_{ub}|$, since the value obtained for the branching ratio $Br(B \to$

$\mu\nu\gamma) \simeq 2 \cdot 10^{-6}$ is reasonably large and, in addition, is theoretically under control in view of the simplicity of the hadronic system.

Note added
After completion of the previous version of this work we became aware of recent papers [23, 24] where a similar technique has been applied to the radiative decays $B \to \rho + \gamma$ and $B \to \omega + \gamma$ and in [24] to the $B \to l\nu\gamma$ decay.

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Figure 1: The normalized differential width $\Delta \equiv 1/\Gamma(d\Gamma/dx)$ of the decay $B \to l\nu\gamma$ as functions of $x = 1 - 2E/m_B$ where E is the photon energy. The solid and dashed curves correspond to the results of the light cone sum rule calculation (Eqs.(25,26)) with the dipole ansatz for the form factor $f(p^2)$ and the quark model calculation of Ref. [7], respectively.

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